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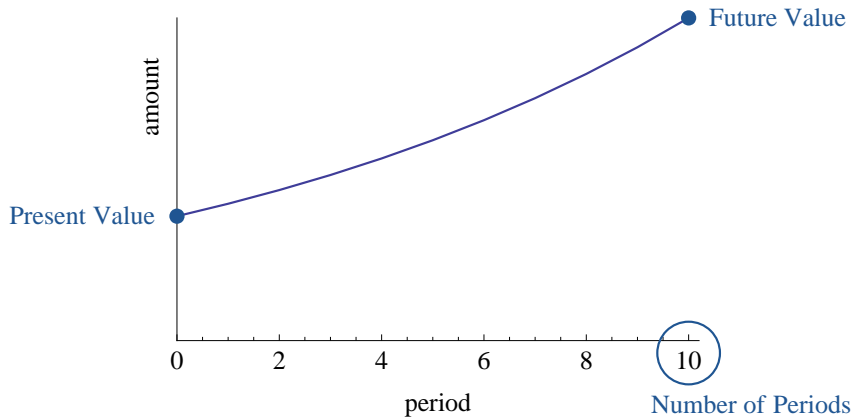
SCHOOL OF BUSINESS

BUSI 2504i - Essentials of Business Finance

Wednesday, January 13, 2010

- §5.1-5.2 review and examples
- §5.3 more on present and future value
- §6.1 future/present value of multiple cash flows
- §6.2 annuities and perpetuities

- §5.1-5.2 - review and examples
- §5.3 - more on present and future value
- §5.4 - summary and conclusions



- FV = future value
- PV = present value
- I/Y = period interest rate (**note: interest is entered as a percent, not a decimal**)
- N = number of periods
- Remember to clear the registers ($CLR TVM$) after each problem

Alexander Industries just had a very profitable year. The owner has decided to invest \$225,000 of the profits in a venture that pays an 8% rate of return for fifteen years. How much more would the investment have been worth if the owner could have made 9% on this investment?

answer: \$105,820.50

Your parents set up a trust fund for you 10 years ago that is now worth \$19,671.51. If the fund earned 7% per year, how much did your parents invest?

- 10
- 7
- 19,671.51
- = -10,000

- For a given interest rate, the longer the time period, the lower the present value
 - **example** - What is the present value of \$500 to be received in 5 years? 10 years? The discount rate is 10%
- For a given time period, the higher the interest rate, the smaller the present value
 - **example** - What is the present value of \$500 received in 5 years if the interest rate is 10%? 15%?

- Often we will want to know what the implied interest rate is in an investment

rearrange FV equation and solve for r

$$FV = PV(1 + r)^t$$

$$\Rightarrow (1 + r)^t = \frac{FV}{PV}$$

$$\Rightarrow 1 + r = \left(\frac{FV}{PV}\right)^{1/t}$$

$$\Rightarrow r = \left(\frac{FV}{PV}\right)^{1/t} - 1$$

very simple to do with BAII Plus calculator

CPT I/Y

"If you are using a financial calculator, the calculator views cash inflows as positive numbers and cash outflows as negative numbers. Be sure and remember the sign convention or you will receive an error when solving for r or t "

You are looking at an investment that will pay \$1,200 in 5 years if you invest \$1,000 today. What is the implied rate of interest?

- 5
- -1,000 (you pay 1000 today)
- 1,200 (you receive 1200 in 5 years)
- = 3.714%

Suppose you have a 1-year old son and you want to provide \$75,000 in 17 years towards his college education. You currently have \$5,000 to invest. What interest rate must you earn to have the \$75,000 when you need it?

- 17
- -5000
- 75,000
- = 17.27%

You want to purchase a new car and you are willing to pay \$20,000. If you can invest at 10% per year and you currently have \$15,000, how long will it be before you have enough money to pay cash for the car?

- 10
- -15,000
- 20,000
- = 3.02 years

- §6.1 - future and present value of multiple cash flows
- §6.2 - annuities and perpetuities

6.1 multiple cash flows: future value - example

You currently have \$7,000 in a bank account earning 8% interest. You think you will be able to deposit an additional \$4,000 at the end of each of the next three years. How much will you have in three years?

Find the value at year 3 of each cash flow and add them together:

- **today (year 0):** 3 ; 8 ; -7000 ; = 8817.98
- **year 1:** 2 ; 8 ; -4000 ; = 4665.60
- **year 2:** 1 ; 8 ; -4000 ; = 4320
- **year 3:** value = 4,000
- **total value in 3 years:**

$$8817.98 + 4665.60 + 4320 + 4000 = 21,803.58$$

6.1 multiple cash flows: present value - example

You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

Find the present value of each cash flow and add them together:

- **year 1:** 1 ; 10 ; 1000 ; = -909.09
- **year 2:** 2 ; 10 ; 2000 ; = -1652.89
- **year 3:** 3 ; 10 ; 3000 ; = -2253.94
- **total present value:**

$$909.09 + 1652.89 + 2253.94 = 4815.93$$

Another way to use the financial calculator for uneven cash flows is to use the cash flow keys:

- Press **CF** and enter the cash flows beginning with year 0
- You have to press **Enter** for each cash flow
- Use the down arrow key **↓** to move to the next cash flow
- The **Fxx** is the number of times a given cash flow occurs in consecutive years

6.1 multiple cash flows: using ba ii plus calculator (cont.)

- Cash flows have to occur at even intervals, so if you skip a year, you still have to enter a zero cash flow for that year
- Use **NPV** to compute the present value by entering the interest rate for **I**, pressing the down arrow and then **CPT**
- Can double check numbers entered by using **↑** and **↓**
- Clear the cash flow keys by pressing **CF** and then **CLR Work**

6.1 multiple cash flows: present value - previous example

You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

Instead you can use the CF (cash flow) feature:

- CF (display now shows “CF₀=”)
- “CF₀ = 0” ↓ (1st cash flow occurs in year 1, not year 0)
- “C01=” 1000 ENTER; ↓
- “F01= 1” ↓ (\$1000 cash flow occurs only once, ie. frequency is 1)

6.1 multiple cash flows: present value - previous example (cont.)

- “C02=” 2000 ;
- “F02= 1”
- “C03=” 3000 ;
- “F03= 1”
- ; (display now shows “I = ”)
- 10 ; (display now shows “NPV = ”)
- = 4815.93 (same answer as before)

- **Annuity** - finite series of equal payments that occur at regular intervals
 - If the first payment occurs at the end of the period, it is called an **ordinary annuity**
 - If the first payment occurs at the beginning of the period, it is called an **annuity due**
- **Perpetuity** - infinite series of equal payments

annuity formulas

$$FV = PMT \left(\frac{(1+r)^t - 1}{r} \right)$$

$$PV = PMT \left(\frac{1 - \frac{1}{(1+r)^t}}{r} \right)$$

perpetuity formula

$$PV = \frac{PMT}{r}$$

“**living off interest**”: How much do you need to invest today at 5% to receive \$100,000 per year forever? *this is a perpetuity...*

answer:
$$PV = \frac{\$100,000}{.05} = \$2,000,000$$

Instead, how much do you need to invest today at 5% to receive \$100,000 per year for 20 years with no money left in the account at the end of 20 years? *this is an annuity...*

answer: 20 N ; 5 I/Y ; 100,000 PMT ; 0 FV ; CPT PV = -1,246,221

Suppose you begin saving for your retirement by depositing \$2,000 per year in an RRSP. If the interest rate is 7.5%, how much will you have in 40 years?

- 40
- 7.5
- -2000
- = 454,513.04

After carefully going over your budget, you have determined that you can afford to pay \$632 per month towards a new sports car. Your bank will lend to you at 1% per month for 48 months. How much can you borrow?

- 48
- 1
- -632
- = 23,999.54

You ran a little short on your February vacation, so you put \$1,000 on your credit card. You can only afford to make the minimum payment of \$20 per month. The interest rate on the credit card is 1.5% per month. How long will you need to pay off the \$1,000?

- 1.5
- 1000
- -20
- = 93.111 months (= 7.76 years)

Suppose you borrow \$10,000 from your parents to buy a car. You agree to pay \$207.58 per month for 60 months. What is the monthly interest rate?

- 60
- 10,000
- -207.58
- = .75%

- The perpetuities discussed so far are annuities with constant payments
- Growing perpetuities have cash flows that grow at a constant rate and continue forever

growing perpetuity formula
(r : interest rate, g : growth rate)

$$PV = \frac{PMT_1}{r - g}$$

Your recently departed rich, eccentric uncle has left for you in his will a large sum of money. Unfortunately, rather than give you this sum of money immediately, he has instructed the executor of the will to pay you \$10,000 in one year. This payment is to grow by 9% each year and to be made each year forever. If the appropriate discount rate is 10%, how much have you actually inherited?

answer: \$1,000,000

Hoffstein Corporation is expected to pay a dividend of \$3 per share next year. Investors anticipate that the annual dividend will rise by 6% per year forever. The required rate of return is 11%. What is the price of the stock today?

answer: \$60.00

- Growing annuities have a finite number of growing cash flows

growing annuity formula
(r : interest rate, g : growth rate)

$$PV = \frac{PMT_1}{r - g} \left[1 - \left(\frac{1 + g}{1 + r} \right)^t \right]$$

Gilles Lebouder has just been offered a job at \$50,000 a year. He anticipates his salary will increase by 5% a year until his retirement in 40 years. Given an interest rate of 8%, what is the present value of his lifetime salary?

answer:

$$PV = \frac{\$50,000}{.08 - .05} \left[1 - \left(\frac{1.05}{1.08} \right)^{40} \right] = \$1,126,571$$